Solution.—Take the given point as the origin, let the diameter of the circular field be represented by (a); put θ for any angle of elevation and ψ for the angle of azimuth so taken that when the projectile will fall on the circumference of the field we shall have $\psi = 2\theta$. Now since any portion of the surface of a hemisphere whose radius is (a) (the diameter of the given circle) and whose center is at the given point is expressed by

$$a^2 \int\int \cos\theta \, d\,\theta \, d\,\psi,$$

therefore the favorable cases will be expressed by the integral

$$a^2 \int_0^{2\theta} d\,\psi \int_0^{\frac{\pi}{4}} \cos\theta \, d\,\theta,$$

and this divided by $\frac{\pi a^2}{\sqrt{2}}$ will give the chance required; therefore we have

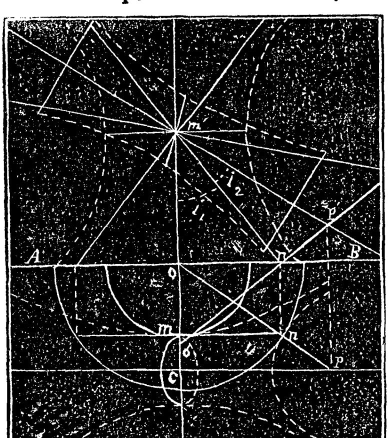
$$\frac{a^2 \int_0^{2\theta} d\,\psi \int_0^{\frac{\pi}{4}} \cos\theta \, d\,\theta}{\frac{\pi\, a^2}{\sqrt{2}}} = \frac{1}{2} - \frac{2}{\pi}(\sqrt{2} - 1)$$

for the chance that the projectile will fall on the circular field.

TANGENCY OF HYPERBOLOIDS OF REVOLUTION.

BY PROF. C. M. WOODWARD, ST. LOUIS, MO.

In his Applied Mechanics p. 430, under the head of skew-bevel wheels, Prof. Rankine says: "If two hyperboloids, equal or unequal be placed in the closest possible contact, they will touch each other along one of the generating straight lines of each which will form their line of contact."



This matter of tangency is stated without proper limitation, but the graphical method given later for finding the obliquities and the gorge circles of the required hyperboloids involves the condition of possibility of such a tangency, which I propose to deduce directly from two tangent hyperboloids by the methods of descriptive geometry.

Let r_1 and r_2 be the radii of the two gorge circles, and i_1 and i_2 the

obliquities of the two surfaces (*i. e.* the angles which the elements make with their respective axes); then the required condition is

$$r_1 : r_2 = \tan i_1 : \tan i_2.$$

Take, as in the Fig., the plane through the axis of the first surface, parallel to the axis of the second as the vertical, and a plane perpendicular to the axis of the first surface, as the horizontal plane. Since the surfaces are tangent, and the plane tangent to both at the point where their gorge circles touch each other contains the common element and is parallel to the vertical plane, the common element is parallel to the vertical plane and its vertical projection is a common asymptote to the vertical projections of the surfaces.

At any point of the element of contact, as at N, (n, n') draw a common normal to the two surfaces- It intersects both axes, one in O, and the other in P.

Since $M\,N$ is parallel to vertical plane, $o'\,n'\,p'$ must be perpendicular to $m'\,n'$. Now the horizontal projecting plane of $M\,N$ divids the perpendicular between the axes into r_1 and r_2, and the normal into $O\,N$ and $N\,P$; hence

$$r_1 : r_2 = O\,N : N\,P$$
$$= o'\,n' : n'\,p'.$$

The obliquities of the surfaces are given in full size in the vertical projection; that is $\quad i_1 = o'\,m'\,n'$ and $i_2 = n'\,m'\,p'$.

Hence, $\quad \tan i_1 : \tan i_2 = o'\,n' : n'\,p'$

and $\quad r_1 : r_2 = \tan i_1 : \tan i_2.$

Q. E. D.

SOLUTIONS OF PROBLEMS IN NOS. 3 AND 4.

Solutions of problems in No. 4, have been received as follows: From Prof. W. E. Arnold, 16 and 18; R. J. Adcock, 17; Marcus Baker, 16, 17, 18 & 19; S. J. Child, 16 & 18; George L. Dake, 16; Prof. A. B. Evans, 16, 17, 18 & 19; Henry Gunder, 16, 17, & 18; Prof. E. W. Hyde, 16, 17, & 18; William Hoover, 16; Phillip Hoglan, 18; Prof. I. N. Jones, 16; J. B. Mott, 16; L. E. Newcomb, 16; A. W. Mason, 16; Prof. A. W. Phillips, 16, 17, 18 & 19; L. Regan, 16; John W. P. Reid, 16; Henry A. Roland, 17 & 19; Miss Kittie Robinson, 16; Prof. Selden Sturges, 16, 17 & 18; E. B. Seitz, 16, 17, 18 & 19; S. W. Salmon, 16, 17, 18 & 19;